Sturm's method for inequalities

1. a) Suppose we replace each factor in the product 100·101·102·...·200 with 150. Will the product increase or decrease? The same question for the sum 100+101+...+200.
б) Will the sum $\frac{1}{100}+\frac{1}{101}+\cdots+\frac{1}{199}+\frac{1}{200}$ increase or decrease, if each term be replaced with $\frac{1}{150}$?

2. Let the sum of two positive real numbers *a* and *b* is fixed. Which of the following expressions increase and which decrease if *a* and *b* be moved closer to each other?

a) ab **b)** a^2+b^2 **c)** $\frac{1}{a}+\frac{1}{b}$ **d)** a^4+b^4 **e)** $\sqrt{a}+\sqrt{b}$ **f)** a^n+b^n **g)** $\frac{1}{a^n}+\frac{1}{b^n}$.

Sturm' method. Make a chain of steps, increasing the number of equal terms one by one. If each step increase LHS and decrease RHS, and at last you get LHS=RHS, then originally LHS<RHS.
3 Prove that for all 100-gons inscribed in the circle the regular 100-gon has the greatest area.
4. Prove the inequality for *arithmetic mean*, *geometric mean* and *harmonic mean*:

if $a_{1,}a_{2,}\ldots,a_n>0$ then $\frac{a_1+a_2+\ldots+a_n}{n}\geq\sqrt[n]{a_1a_2\ldots a_n}\geq\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}}$

5. Given 100 pens painted 7 colours. A pair of pens considered as *good* if they are of different colour. Find the maximal number of good pairs.
6. Compare $(1+\frac{1}{x_1})(1+\frac{1}{x_2})\ldots(1+\frac{1}{x_n})$ and $(n+1)^n$ for $x_{1,}x_{2,}\ldots,x_n>0, x_1+x_2+\ldots+x_n=1$
7. $\sqrt{1+4a}+\sqrt{1+4b}+\sqrt{1+4c}+\sqrt{1+4d}$ and $4\sqrt{2}$ for $a,b,c,d>0, a+b+c+d=1$
8. A *n*-gon has the inscribed circle of radius *R*. Find the minimum perimeter of the polygon.

Credit problems

SI1. For *n* positive numbers $a,b,\ldots,g$ and real $p\neq0$ denote $A_p=(\frac{a^p+b^p+\ldots+g^p}{n})^{\frac{1}{p}}$.
Compare A_p and A_q for $p<q$.
SI2. For all polygons inscribed in the circle find the polygon with the greatest ratio of the area to the number of sides.
SI3. Given $x_{1,}x_{2,}\ldots,x_k>0$, $x_1^2+\ldots+x_k^2<\frac{x_1+\ldots+x_k}{2}<\frac{x_1^3+\ldots+x_k^3}{4}$.
a) Prove that $k>50$.
b) Give an example of these numbers for some *k*.
c) Find minimum possible value for *k*.

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