Right or Wrong: Examples in Calculus

Let $f, g, P: \mathbb{R} \to \mathbb{R}$

1. **a)** $f(x)$ is continuous and takes each positive value (i.e. $\forall c>0 \quad \exists x: f(x)=c$). Does that imply $f(x)$ has a zero?
   b) A polynomial $P(x)$ takes each positive value. Does that imply $P(x)$ has a zero?
2. $f(1)=6, f(5)=10$.
   a) $f(x)$ is continuous. Does that imply $6<f(x)<10$ for any x, $1<x<5$?
   b) $f'(x)$ exists for any x. Does that imply $f'(x)=1$ for some x?
3. Let f' be continuous. Are the following propositions right or wrong:
   a) If f has infinitely many zeros then f' has infinitely many zeros.
   b) If f' has infinitely many zeros then f has infinitely many zeros.
4. $f(0)=0, f'(x)$ exists for any x.
   b) Does $|f'(x)|\le 1$ for any $|x|\le 1$ imply $|f(x)|\le 1$ for any $|x|\le 1$?
   a) Does $|f(x)|\le 1$ for any $|x|\le 1$ imply $|f'(x)|\le 1$ for any $|x|\le 1$?
   Definition. $f(x)$ is *even* if $f(-x)=f(x)$ for any x, and is *odd* if $f(-x)=-f(x)$ for any x.
5. **a)** $f(x)$ is even, $f'(x)$ exists for any x. Does that imply $f'(x)$ is an odd function?
   b) $f(x)$ is odd, $f'(x)$ exists for any x and has no zeros. Does that imply $f(x)$ takes each positive value?

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