Coding

Basic coding sets

Given an alphabet with N letters. A *word* is just a final sequence of letters. Let us count all the words of some kind.

(**Words with repetition**) There are N^k different words of length k (i.e. consisting of k letters).

(**Words without repetition**) There are $N(N–1)(N-2)\ldots(N–k+1)$ different words of length k, rconsisting of different letters. The number can also be denoted as $N^{\underline{k}}$ (called N to decreasing power k).

(**Permutations**) Both the alphabet and a word consist of k different letters. There are $k(k–1)(k–2)\ldots2\cdot1 = k!$.

(**Unorded pairs**) There are $N(N-1)/2$ subsets consisting of two different letters. Subsets {a, b} and {b, a} considereres to be the same subset.

1. a) How many five digit numbers consist of odd digits?
b) How many five digit numbers consist of distinct odd digits?
2. A password may consist of 8 symbols, each symbol must be either a digit or english letter (uppercase or lowercase letter considered to be different). What is the total number of passwords?
3. 70 students are ready to take an exam. They should sit in four rooms. A version of placement is a row where one can see in which room each student shall sit. The headmaster asked to make a table with all possible version of sitting. (Versings with some rooms are vacant are also allowed). How many rows has the table?

Coding

A coding makes one-to-one corresponding between objects or situations from a the problem to some combinatorial set. For example, such a correspondence to sequences of 0 and 1 is called a *binary coding*.

4. How man ways are there to place a black, a white and a red queen to the chessboard of size 8x8?
5. How many different subsets has a 10 element set?
6. How many 6 digit numbers have third digit coinciding with **a)** the last digit; **b)** the first digit?
7. How many square tables of size N consist of 0 and 1 and have the sum 1 for each row and each column?
8. A good row consists of two or more subsequent integers, and each integer is a 2 digit number (e.g. (11, 12, 13) or (45, 46, 47, 48, 49) are good rows). Find the total amount of good rows.
9. A 9 digit word begins with M and each of next letters either coincide with the previous letter or is the next letter in English alphabet. What is the total number of such words?

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Credit problems

Cd1. How many ways to represent 2018 as a sum of positive integers which almost equal?

Two numbers considered to be *almost equal* if their difference is not more then 1. If two representations differs with the order of terms only, they considered to be the same.

Cd2. In haw many ways one can place the numbers 1, 2, …, 20 in a row so to each number except 1 being greater then at least one of its neighbors?

Cd3. A flee jumps along number axis. It starts from a point strictly between 0 and 1. Before every jump the flee measure the distance to the closest integer at the left and jumps this distance to the right. After the 17^{th} jump the flee first time came on an integer, namely om 15. Find the number of possible start points.

Cd4. There are N points, each two are connected with a segment. It is allowed to choose a direction for each segment and travel from point to point only following directions. The set of directions must satisfy the Condition: rom any point one can either not leave or can leave but can not come back. In how many ways one can chose the set of directions satisfying the Condition?

Cd5. In how many ways one can divide a big rectangle 2x12 into smaller rectangles of size 1x2?