Prime Factorization: Construct using primes

GCD – greatest common divisor, LCM – least common multiple, *a*, *b*, *c*, *d*, *n* – positive integers. A *perfect square* is a square of a positive integer (e.g 4, 49, 100).

1. a) Give an example of three positive integers $a<b<c$ such that GCD(a, b)>GCD(b, c).
b) Give an example of a strictly increasing sequence of integers $n_{1,} n_{2,} \dots, n_9$ such that the sequence $GCD(n_{1,} n_2), GCD(n_{2,} n_3), \dots, GCD(n_{8,} n_9)$ is strictly decreasing.
2. a) Give an example of three positive integers such that none of them is a divisor of another, but each of them divides the product of two others.
b) Give an example of ten such integers.
3. a) Place 4 integers at the vertices of quadrangle such that
(one of the integers be a divisor of an other <=> both integers be on the same side).
b) Can one place 8 integers at the cube vertices such that
(one of the integers be a divisor of an other <=> both integers be on the same edge).
4. Remove one of 100 factors from the product $1!\cdot 2!\cdot \ldots \cdot 100!$ to get a perfect square.

Credit problems

PF1. Find the least positive integer *N* such that *N*/2 is a perfect square, *N*/3 is a perfect cube and *N*/5 is a perfect 5th power.

PF2. Can *n*! have exactly 5 zeroes at the end?

PF3. Prove that **a)** GCD(a, b)LCM(a, b)=ab;
b) GCD(a, b, c)LCM(ab, ac, bc) = abc
c) LCM(a, b, c) GCD(ab, ac, bc) = abc

PF4. Can one place 9 integers at the 9-gon vertices such that
(one of the integers be a divisor of an other <=> both integers be on the same side).

PF5. Does there exist a strictly increasing sequence of integers $n_{1,} n_{2,} \dots, n_9$ such that the sequence $LCM(n_{1,} n_2), LCM(n_{2,} n_3), \dots, LCM(n_{8,} n_9)$ is strictly decreasing?
PF6. Integers from 2 to 10001 are stored in 10000 cells. One knows GCD for any two cells. Is it enough to find out the number in each cell?