What is possible and what is impossible for quadratic polynomials?

Let $Q(x)=ax^2+bx+c$, where $a\neq 0$, be a quadratic with real coefficients.

1. Can all the values of $Q(x)$ be positive if $b>a+c$?
2. **a)** For each even integer x the value $Q(x)$ is an integer. Can value for $Q(x)$ be noninteger for some odd integer?
   b) For each positive integer x the value $Q(x)$ is an integer. Can value for $Q(x)$ be noninteger for some negative integer?
3. Can the whole graph of the equation $y=Q(x)$ lie higher than the graph of the equation $y=x^4$?
4. Given are 2 reduced quadratic polynomials. Can their graphs divide the plane into more then 4 parts? (A quadratic Q is *reduced* if $a=1$).
5. **a)** Can equation $Q(x)=\sqrt{x}$ have more then 2 roots *?*
   b) Can equation $Q(x)=\sqrt{x}$ have more then 4 roots?
6. **a)** Can all the values $Q(1)$, $Q(2)$, $Q(3)$, …, $Q(40)$ be different prime numbers?
   b) Can all the values of $Q(x)$ for x integer be prime numbers?

Credit problems

QP1. One increased both coefficients of the equation $x^2 + px + q = 0$ by 1 and got a new equation. Repeating the operation 8 times more one got 8 extra equations. For each of 10 equations, can both roots be integers?

QP2. Given are 10 quadratic polynomials, each has two roots. For any two of the polynomials consider a new polynomial equal to their sum. Can it happen that each of the new polynomials has no roots at all?

QP3. Call a positive integer *unitary* if all digits in its decimal expression are 1 (e.g. 1, 111, 1111 are unitary). Is there Q such that for any unitary x, the value $Q(x)$ is also unitary?

QP4. Given are integers p and q such that for any integer x the value $Q(x)=x^2+px+q$ is positive. Can $Q(x)$ be negative for some noniteger x*?*

QP5. In the equation $x^2+px+q=0$, both coefficients were changed (increased or decreased) by less than 0,001. Can the larger root of the equation be changed with more than 1000?

QP6. Can all the values of $Q(x)$ be rational for any rational x and irrational for any irrational x?

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