

Test 1 (2018)

1. Given an infinite sequence of digits 1234567891011121314... On what places there first occur subsequence ...2018...?
2. Given two function graphs $y=ax^3+bx^2+cx+d$ and $y=ex^3+fx^2+gx+h$ where 8 coefficients are all distinct. Find the minimal number of their common points.
3. Given a matrix M of size 2×2 where for each element holds $|m_{ij}| \leq 5$. Find the maximum possible value for $\det(M)$.
4. A function $f(x)$ has 10 roots, its derivative $f'(x)$ is continuous and has 3 common roots with $f(x)$. What minimal number of local maximums $f(x)$ can have?
5. z and w are two complex numbers, $|z+w|=2$, $|z-w|=5$. Find the maximal possible value for $|z^2+w^2|$.
6. Straight lines are drawn via point $(1, 1)$. They divide the plane into 9° angles. One line is parallel to x -axis. Some of the lines intersect the extra line $y = 11 - x$. Find the sum of x -coordinates for all intersection points.
7. Find the total number of 10-digital positive integers with the sum of digits equal to 10.
8. There are two square lawns $ABCD$ and $ADEF$, there boarders are walkways. Mr Watson walks around $ABCD$ clockwise with a constant speed, Mr Holmes walks around $ADEF$ clockwise with a constant speed 20% higher then mr. Watson. They meet each other sometimes on the common walkway AD . First time they met at 10.00 am, the second time at 10.10, the third time at 10.20. When they met the 4th time?
9. Some days in July were rainy, some not. A period can consist of one or more consecutive days. What is maximum possible number of periods, which includes an odd number of rainy days?
10. A positive integer N can be transformed to the exponent by moving the last digit up (e.g., number 179 is transformed to 17^9 , or 2007 transformed to 200^7). Find the maximal N for which the transformed number is divisible by N .