What is possible and what is impossible for a quadratic polynomial?

Let $Q(x)=ax^2+bx+c$, where $a\neq 0$, be a quadratic with real coefficients.

1. **a)** Can the sum of coefficients of $Q(x)$ be positive while some of its values are negative?
   b) Can all the values of $Q(x)$ be positive if the sum of coefficients of $Q(x)$ is negative?
2. For any integer x the value $Q(x)$ is an integer.
   a) Can all the coefficients of $Q(x)$ be noninteger?
   b) Can some of the coefficients of $Q(x)$ be noninteger?
3. Can the whole graph of the equation $y=Q(x)$ lie higher than the graph of the equation $y=x^3$?
4. Given are 2 reduced quadratic polynomials. One of them has two distinct roots greater than 1000, the other one has two distinct roots smaller than 1000. Can the sum of these polynomials have one root greater than 1000 and the other root less than 1000?
   (A quadratic Q is *reduced* if $a=1$).
5. Can the equation $Q(x)=\sin x$
   a) have exactly 1000 roots?
   b) have infinitely many roots?
6. One increased both coefficients of the equation $x^2 + px + q = 0$ by 1 and got a new equation. Repeating the operation 8 times more one got 8 extra equations. For each of 10 equations, can both roots be integers?
7. Given are 10 quadratic polynomials, each has two roots. For any two of the polynomials consider a new polynomial equal to their sum. Can it happen that each of the new polynomials has no roots at all?
8. **a)** Can all the values $Q(1)$, $Q(2)$, $Q(3)$, …, $Q(40)$ be different prime numbers?
   b) Can all the values of $Q(x)$ for x integer be prime numbers?
9. Call a positive integer *unitary* if all digits in its decimal expression are 1 (e.g. 1, 111, 1111 are unitary). Is there Q such that for any unitary x, the value $Q(x)$ is also unitary?
10. Given are integers p and q such that for any integer x the value $Q(x)=x^2+px+q$ is positive. Can $Q(x)$ be negative for some noninteger x?
11. In the equation $x^2+px+q=0$, both coefficients were changed (increased or decreased) by less than 0,001. Can the larger root of the equation be changed with more than 1000?
12. Can all the values of $Q(x)$ be rational for any rational x and irrational for any irrational x?

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