5 problems for the 1st homework

1. The positive integer n can be replaced by ab if $a + b = n$, where a and b are positive integers. Can the number 2001 be obtained from 22 after a sequence of such replacements?

2. In three piles there are 51, 49, and 5 stones, respectively. You can combine any two piles into one pile or divide a pile consisting of an even number of stones into two equal piles. Find the maximal number of piles that can be obtained fter a sequence of such operations?

3a. There are 5 identical paper triangles on the table. Each can be moved in any direction parallel to itself (i.e., without rotating it). Is it true that then any one of them can be covered by the 4 others?
3b. There are 5 identical equilateral paper triangles on the table. Each can be moved in any direction parallel to itself. Prove that any one of them can be covered by the 4 others in this way.

4. Several boxes are arranged in a circle. Each box may be empty or may contain one or several chips. A move consists of taking all the chips from some box and distributing them one by one into subsequent boxes clockwise starting from the next box in the clockwise direction.
(a) Suppose that on each move (except for the first one) one must take the chips from the box where the last chip was placed on the previous move. Prove that after several moves the initial distribution of the chips among the boxes will reappear.
(b) Now, suppose that in each move one can take the chips from any box. Is it true that for every initial distribution of the chips you can get any possible distribution?

5. Find at least one polynomial $P(x)$ of degree 2001 such that $P(x) + P(1 - x) = 1$ holds for all real numbers x.