Test 2

1. Prove that $\quad 1 + \dfrac{1}{2} + \ldots + \dfrac{1}{n} \geq \ln(n) \quad \forall\, n \in \mathbb{N}$

2. Is it possible that LCM(a, b) = LCM($a+c$, $b+c$) for positive integers a, b and c?

3. Prove that any power of the polynomial $x^4 + x^3 - 3x^2 + x + 2$ has at least one negative coefficient.

4. Does there exist an infinite sequence of real numbers such that the sum of every ten successive numbers is positive, while for every n the sum of the first $10n+1$ successive numbers is negative?