Diophantine Equations and Factorization

Theory. When a product of expressions is equal to an integer, one can split the equation. Factorize the integer into as much factors as there are expressions and build the equation system where each expression is equal to one factor. As the correspondence between expressions and integer factors is not unique, one must go through all possible factorization and factor permutations.
For example, the diophantine equation (x+y)y=2 can be reduced to the four following linear systems:

$$\begin{cases} x+y=1 \\ y=2 \end{cases} \begin{cases} x+y=2 \\ y=1 \end{cases} \begin{cases} x+y=-1 \\ y=-2 \end{cases} \begin{cases} x+y=-2 \\ y=-1 \end{cases}$$

Of course, usually it is your task to factorize expressions.

DE1. Find all integer solutions to the equations **a)** $x^2–y^2=28$ **b)** $xy=x+y+3$ **c)** $x^2–y^2= x+y+2$

DE2. Using both remainder technique with factorization, find
a) all positive integer solutions **b)** all integer solutions
of the equation $3^m+7=2^n$.

DE3. Find all integer solutions of the equation $3\cdot 2^m+1=n^2$.

DE4. 100 weights of 1, 2, …, 100 g are placed at the two cups of scales so that the scales are in balance. Prove that one can remove two weights from the left cup and two weights from the right cup and still keep the balance.