Right or Wrong: Examples in Calculus

Let $f, g, P: \mathbb{R} \to \mathbb{R}$

1. . Does $\int_{a-1}^{a} f(x)\,dx = 0 \quad \forall a \in \mathbb{R}$ imply $f(x) = 0 \quad \forall x \in \mathbb{R}$?
2. **(IMC 2009.1.1)** Does $f(r) \le g(r) \quad \forall r \in \mathbb{Q}$ imply $f(x) \le g(x) \quad \forall x \in \mathbb{R}$ if
   a) f and g are non-decreasing?
   b) f and g are continuous?
3. Let $f(0) = 0$.
   a) Does $|f'(x)| \le 1 \quad \forall x \le 1$ imply $|f(x)| \le 1 \quad \forall x \le 1$?
   b) Does $|f(x)| \le 1 \quad \forall x \le 1$ imply $|f'(x)| \le 1 \quad \forall x \le 1$?
4. **(IMC 2007.2.1)** Let f be continuous and $\forall c > 0$ the graph of f can be moved to the graph of cf using only a translation or a rotation. Does that imply that $f(x) = ax + b$ for some $a, b \in \mathbb{R}$?
5. **(IMC 2006.1.1)** Prove or disprove each of following statements
   a) If f is continuous and $range(f) = \mathbb{R}$ then f is monotonic.
   b) If f is monotonic and $range(f) = \mathbb{R}$ then f is continuous.
   c) If f is monotonic and f is continuous then $range(f) = \mathbb{R}$.
6. Let f' be continuous. Prove or disprove each of following statements
   a) If f has infinitely many zeros then f' has infinitely many zeros.
   b) If f' has infinitely many zeros then f has infinitely many zeros.
7. Prove or disprove each of following statements
   a) if f is continuous and takes each positive value then f has a zero.
   b) if the polynomial P(x) takes each positive value then P has a zero.
   c*) if the polynomial $P(x,y)$ takes each positive value then P has a zero.

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