

## How could it happen?

### Invent via analysis

1. A water-melon has been cut into four parts and eaten. Nobody has bitten the rind, but there are 5 water-melon rinds. How could it happen?
2. Positive integers  $a, b, c$  are odd. Neither of them is a perfect square. Could the product  $a^b b^c c^a$  be a perfect square?
3. Alice, Bob and Carl participated in 10 competitions. At least 6 times Alice has shown a better result than Bob done. At least 6 times Bob has shown a better result than Carl done. At least 6 times Carl has shown a better result than Alice done. How could it happen?
4. Is it possible to cut some triangle into 4 convex parts – a triangle, a quadrilateral, a pentagon and a hexagon?

### To understand how it could happen and to use it.

5. a) Does there exist a positive integer  $n$  such that  $n$  and  $n+2009$  has the same sum of digits?  
b) The same question for  $n$  and  $n+333$ ?  
c) Consider all positive five-place integers  $n$  such that  $n$  and  $n+9$  has the same sum of digits. Find the lower-bound estimate for the total amount of such integers.
6. A hunter goes hunting wild ducks everyday, and after coming back he used to say “I have killed more ducks today than before yesterday but less ducks than exactly a week ago”.  
a) Can those words be truthful seven days in a row?  
b) What is the maximal number days in a row for those words to be a truth?
7. In a quadrilateral  $ABCD$   $\angle A=85^\circ$ ,  $\angle B=115^\circ$ ,  $AD=BC$ . Perpendicular bisectors to the segments  $AB$  и  $CD$  intersect each other into the point  $M$ . Find  $\angle MAB$ .