

## **Combinations, logic and geometry**

1. Alex is less than 60 years old. His 5<sup>th</sup> birthday was Tuesday. What weekday was his 45<sup>th</sup> birthday?
2. Jack has written numbers from 1 to 2014 on a blackboard. Let  $a$  be the number of digits 1 on the blackboard and  $b$  be the number of digits 3. Find  $a-b$ .
3. A checkered square  $30 \times 30$  is given. One cuts it into pieces along each diagonal of each square. How many pieces one will get? (On the picture you can see a checkered square  $2 \times 2$ . If one cuts it in the same way one will get 12 pieces.).
4. Numbers 1, 2, 3, ..., 1004 were written in a row. Then the order of the numbers had been changed: first even numbers were written in the ascending order (i.e. 2, 4, 6, 8, ...), then those from the rest which were divisible by 3 ( i.e. 3, 9, 15, ...) again in the ascending order, then in the same way those divisible by 5, divisible by 7 and so on. The last written number was number 1. Find the least number that didn't change its place.
5. The walls of a big room make a rectangular box (see the picture), the sizes are known, and the position of a switch S and a lamp L are also given. Connect S and L with the shortest cable. The cable can run along the walls, the floor and the ceiling in any direction, but can't hang in the air. Give the answer rounded to cm.
6. In the opening ceremony of the Olympics participated less than 2014 sportsmen. Exactly  $1/99$  of them were recordsmen. All participants were arranged in rectangle. It happened that more than 44% of the rows contained at least one recordsman, and the same is true for more than 44% of the columns. How many sportsmen participated at the opening ceremony?

